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Artificial viscosity [Q] has been used for over 40 years[1] to simulate numerically the propagation of waves in a discretized continuum. It is composed typically of terms that are quadratic and linear in the gradient of the particle velocity, and "switches" (numerical representations of physical criteria) for turning the terms *on* and *off*. The quadratic term is active only in the region of a sharp discontinuity, *e.g.*, a shock, where the velocity gradient is large, and spreads (smooths) the discontinuity over a few computational zones, or a fixed length. The linear term damps oscillations resulting from computed timesteps that are Courant stable, but mismatched to the mesh size. The linear Q is active throughout the mesh and consequently can be very dissipative. That a timestep mismatch gives rise to oscillations, for which a Q is required can be demonstrated by noting that elastic uniaxial strain waves (*e.g.* square waves!) can be propagated without Q, attenuation, or dispersion, through a uniform (one and only one fundamental timestep) mesh, if the period of the pulse is an integral number of timesteps. A small change in the period of the pulse creates oscillations, which must be damped. In general, waves are propagated numerically through different materials and nonuniform meshes with various boundary conditions, resulting in a different timestep in each computational zone, so that a linear Q is necessary.

We are interested in simulating the propagation of elastic waves, for which only the linear Q is important. The dispersion and attenuation due to the linear Q can dominate the character of the wave propagation.[2] Consequently, an analytic solution is necessary to assess the quality of various Q-formulations. Blake[3] obtained an analytic solution for the propagation of a spherical wave driven by an exponentially decaying pressure applied to the inside surface of a hollow elastic sphere. This is an ideal problem for comparing different formulations of the linear Q. Its physical characteristics are quite similar to waves generated by underground explosions; thus, the Blake solution is relevant to many real applications.

We will show that Blake's analytic solution can be approximated numerically very well, using a standard tensor linear-Q. The key improvement we make is a new formulation of the switch that turns the Q *on* and *off*, which is in contrast to most of the previous work which has focussed on altering only the functional form of the artificial viscosity. Our improvement requires little additional computational overhead beyond incorporating the tensor linear Q, and thus, is very fast computationally.

The linear Q used in many hydrocodes may be expressed as

$$Q_{ij} = a\eta v_{i,j} = a\eta\left[v'_{i,j} + \frac{1}{3}v_{k,k}\delta_{ij}\right], \tag{1}$$

where $v_{i,j}$ are components of the velocity gradient ($v_{k,k} = \nabla \bullet v$), η is a function of the local sound speed, the zone size, and the density, and a is a user-specified multiplicative constant. The right side of Eq. (1) is a decomposition of the viscosity into deviatoric (primed) and scalar components (δ_{ij} is the Kronecker delta). The deviatoric term is ignored often, so that only a scalar Q remains. Although a scalar Q is fine for hydrodynamic waves, the tensor form is more appropriate for damping oscillations in elastic waves, which can have large deviatoric components.

The typical switch used with Eq. (1) is Q_{ij} *on* if $\nabla \bullet v < 0$, *i.e.*, if the material is compressing volumetrically. Our "modified" formulation of the linear Q, which we denote as Q^m, is obtained by specifying a new switching function for Eq. (1). We write

$$Q^m_{ij} = a\eta v_{i,j}[f_1 + f_2], \tag{2a}$$

$$f_1 \equiv \max\left[\operatorname{sgn}\left[-\sigma_{ij}v_{i,j}\right], 0\right], \text{and} \tag{2b}$$

$$f_2 \equiv \max\left[\operatorname{sgn}\left[\frac{\partial^2}{\partial t^2}(-\sigma_{ij}v_{i,j})\right], 0\right], \tag{2c}$$

where σ_{ij} are the components of the stress tensor (compressive stresses are negative), and $\operatorname{sgn}(x) = 1$ if $x > 0$, -1 if $x < 0$, or 0 if $x = 0$. Thus, f_1 and f_2 equal 0 or 1. The scalar $\sigma_{ij}v_{i,j}$ is the power per unit volume, so that f_1 and f_2 have simple physical interpretations. Figure 1 shows schematically the hypothetical response of a computational mesh to the passage of a uniaxial strain wave. Particle velocity is plotted as a function of distance. The dashed line is an idealized solution for a propagating step wave (compressive or tensile). The solid line is the oscillatory response that would be expected from a numerical solution without Q. The figure also shows the regions of the wavefront in which f_1 and f_2 would be active initially. f_1 is active for compressed regions that are expanding, or for expanded regions that are compressing. f_1 is activated initially only after the peak of the wave, consequently, it does not damp the initial overshoot. f_2 damps the overshoot, because it is constructed from the second derivative of the power per unit volume, so that it "anticipates" the overshoot. The figure shows that f_2 is activated initially at the midpoint of the wavefront. Although f_2 is defined by a temporal derivative, we have used the approximation $\frac{\partial}{\partial t} \to c\frac{\partial}{\partial x}$, where $c \sim c_L$, to compare and contrast in the same figure the regions of initial activity of f_1 and f_2.

Equations (2) were added to DYNA2D,[4] which is a 2D Lagrangian hydrocode. The Blake problem was modelled using a spherical 1D mesh with 100 radial 0.2m thick zones. The driving pressure, P, on the inner surface, which was at a radius of 10m, was $P(\text{GPa}) = 0.1 \exp[-1000t]$, where t is the time in seconds. The density, bulk modulus, and shear modulus of the elastic material were 2000kg/m^3, 36(GPa), and 12.5(GPa). The resulting longitudinal sound speed, c_L, is 5.13m/ms. Figures 2–4 show the velocity profile at 3ms for the analytic solution (dashed line) and three formulations of the linear Q (solid line) listed in Table I. For each of the three cases, the value of the multiplicative coefficient, a, was chosen so that the calculated peak velocity would equal the analytic peak velocity.

The scalar and tensor viscosities shown in Figs. 2 and 3 create many oscillations behind the peak. The numerical results shown in Figs. 2 and 3 are improved if the Q is always *on*, instead of *on* only in compression. However, leaving the Q always *on* introduces too much dissipation, for most realistic problems. This is why the compression only condition has become the "industry standard." Figure 4 shows that the solution using the modified Q is nearly free of oscillations and more closely approximates the analytic solution than the other cases. In addition, we examined several other test problems, other forms of Q, and other switches. Q^m produced results that equalled or exceeded those results in every case.

The purpose of this note was to demonstrate that the accuracy of numerical simulations of wave propagation that use linear artificial viscosity can be improved significantly by altering the traditional activation criteria for the viscosity. Our activation criteria are based on the power per unit volume. Other criteria that yield more accurate results may certainly exist.

Figure #	Viscosity Type	Activity	a
2	Eq. (1): scalar $[v'_{i,j} = 0]$	compression only	0.015
3	Eq. (1): tensor	compression only	0.013
4	Eq. (2): modified	f_1 and f_2 [Eqs. (2b,c)]	0.032

Table I: Parameters for the numerical simulations

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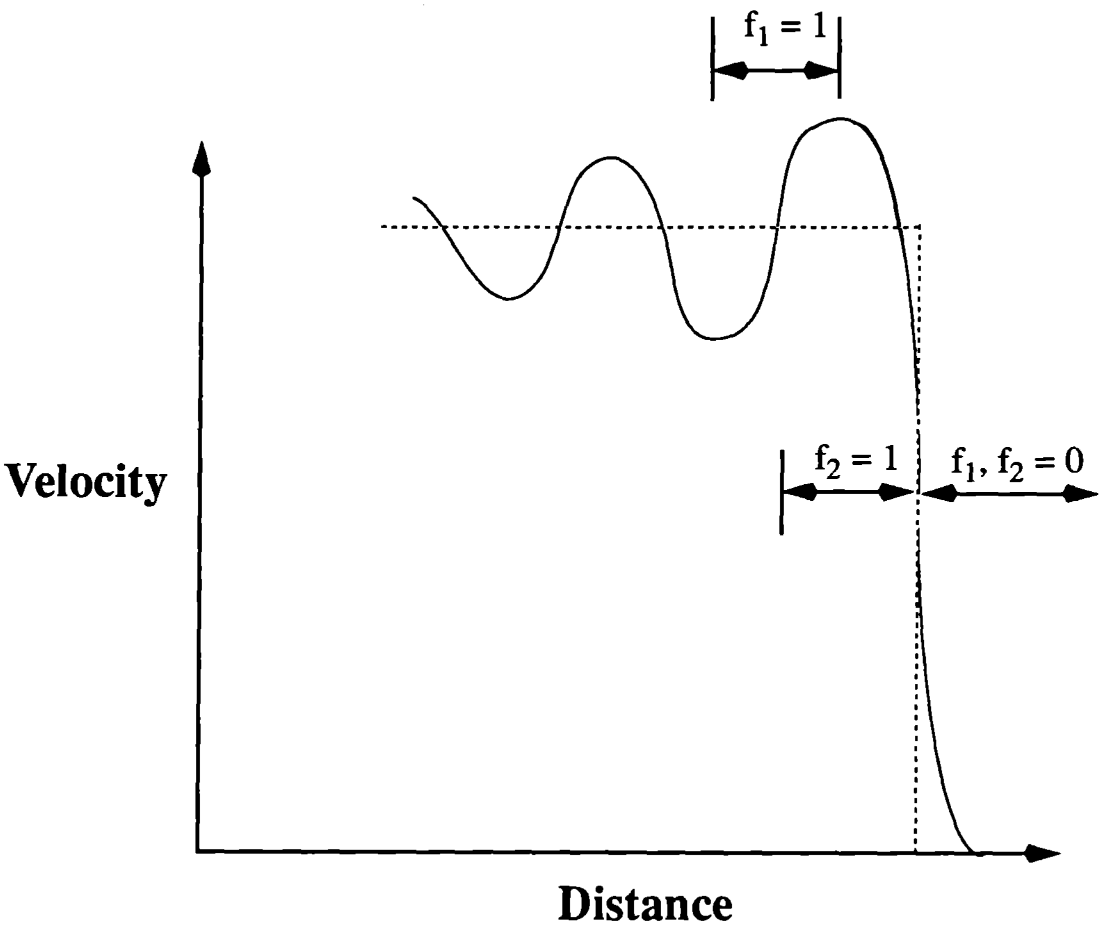

Figure 1– Schematic response of a computational mesh to the passage of a uniaxial strain wave. The particle velocity is plotted as a function of distance, for the hypothetical numeric (solid line) and exact (dashed line) solutions. The regions are shown where f_1 and f_2 are active initially.

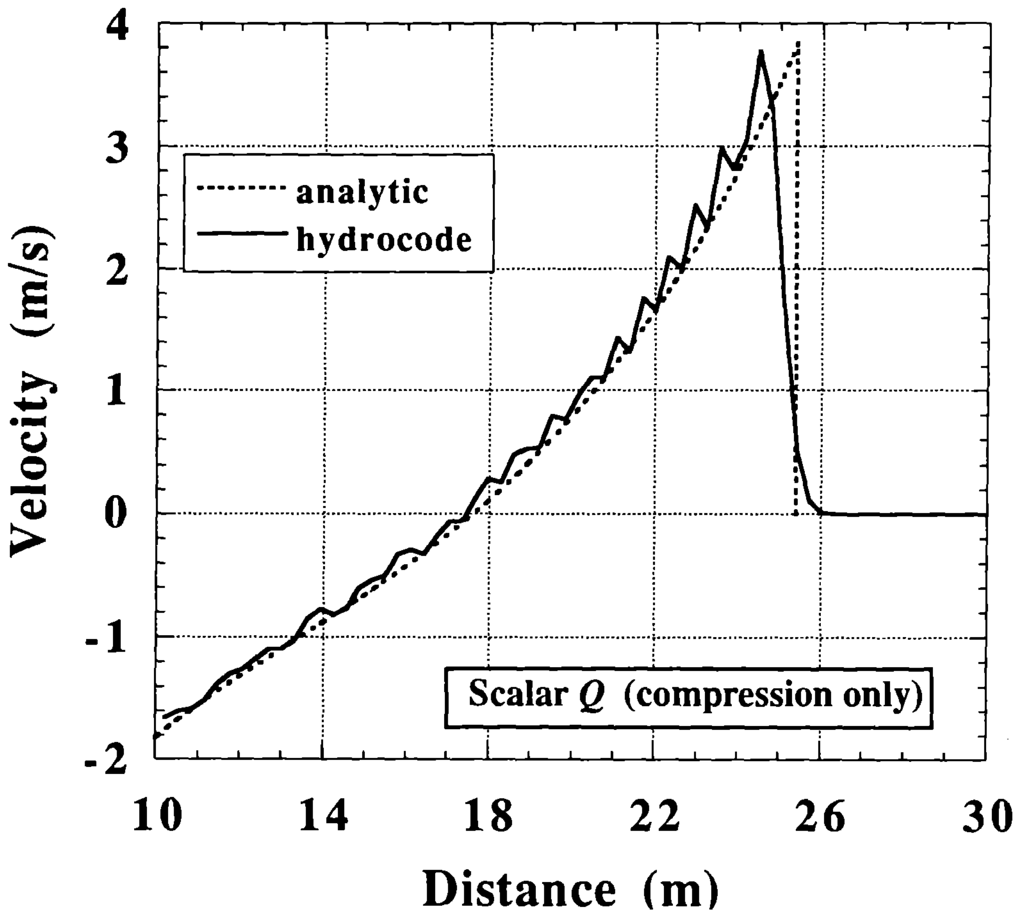

Figure 2– Velocity as a function of distance at 3ms for the analytic (dashed) and the numeric (solid) solutions. The multiplicative constant, a, is 0.015.

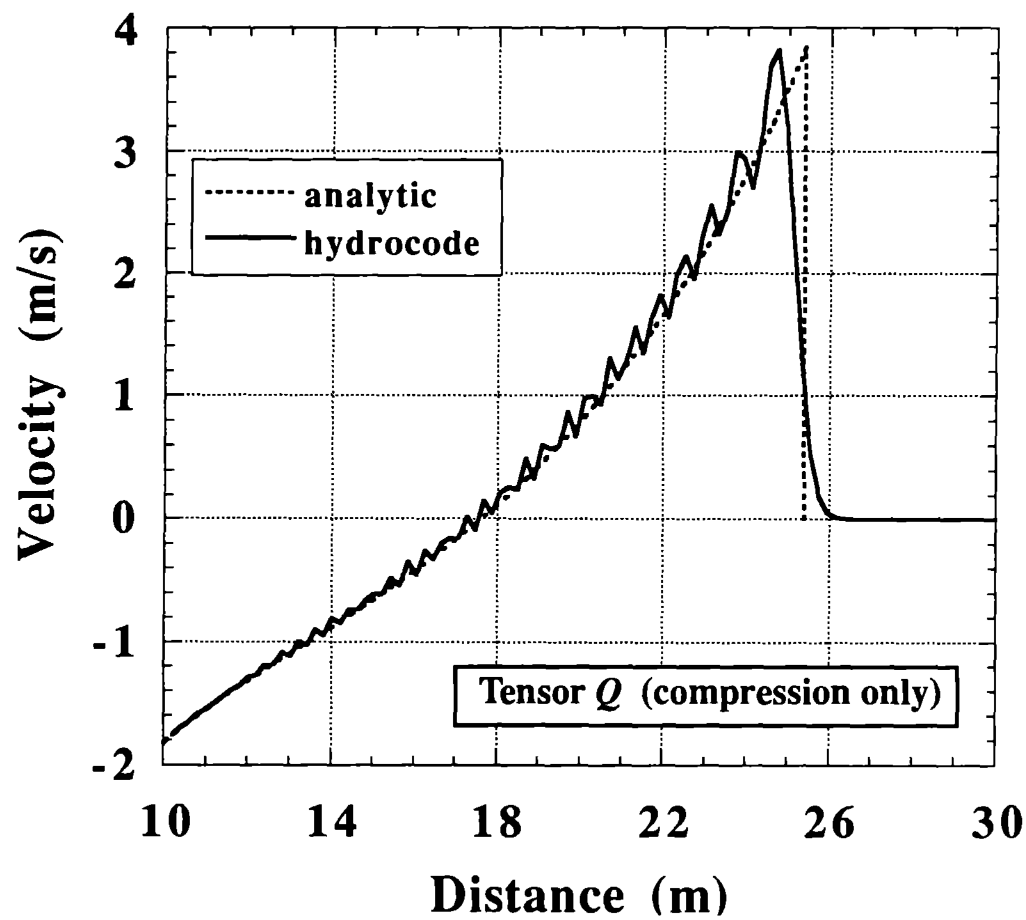

Figure 3– Velocity as a function of distance at 3ms for the analytic (dashed) and the numeric (solid) solutions. The multiplicative constant, *a*, is 0.013.

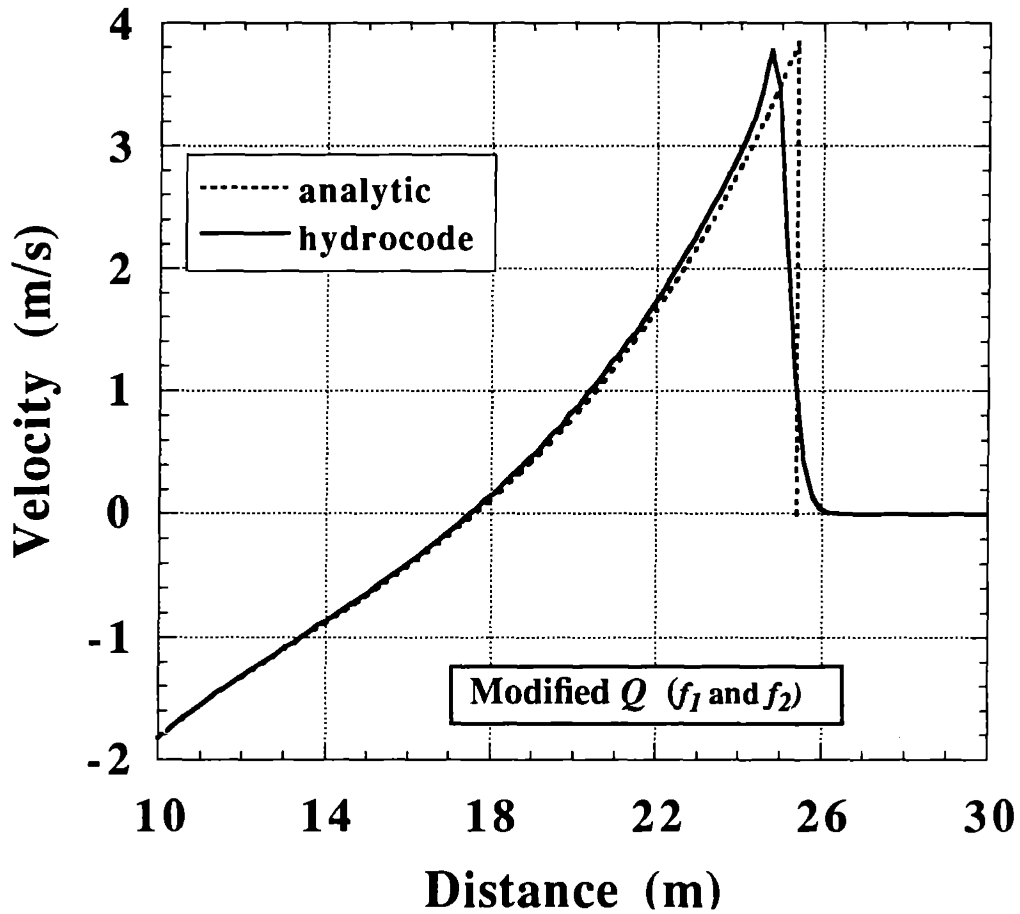

Figure 4— Velocity as a function of distance at 3ms for the analytic (dashed) and the numeric (solid) solutions. The multiplicative constant, a, is 0.032.